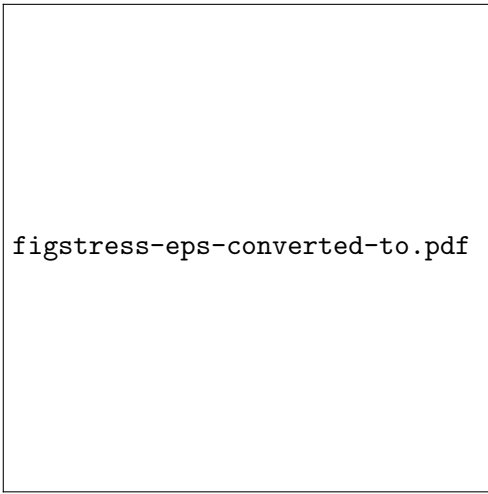


Problem 4 is a simple exercise in using the Maxwell stress tensor to calculate a force. Problem 5 does something similar, but is more challenging. Problem 6 use the Levi-Civita to proof part of the derivation of the conservation law of angular momentum. Problems 4 and 5 are the important problems.

On brightspace (below the table) you can find an additional problem. It is about using the idea of the Green's function, but for a different differential equation. It is also a good exercise in using Fourier transforms. It goes beyond the material required for this course. But can be fun if you are interested...

Problem 4 We have two point charges, but they have opposite sign. We put charge q at $z = b$ and charge $-q$ at $z = -b$. Coulomb tells us that the force of mutual attraction is $q^2/(2b)^2$. Here you are asked to calculate this force by integration of Maxwell's stress tensor, over the xy -plane.

Problem 5 Given an infinitely long wire with radius R , and a homogeneous current density \mathbf{J} , running through the wire in the direction of the positive z -axis.



figstress-eps-converted-to.pdf

- (a) Calculate \mathbf{E} and \mathbf{B} . Use that in the wire $\mathbf{J} = \sigma \mathbf{E}$; σ is the conductivity. [Note there is no need to calculate \mathbf{E} outside the wire, but you can guess what it should be. There is also no real need to calculate \mathbf{B} outside the wire... because in (b) we can use that the fields are continuous at the surface (why?).]
- (b) Calculate, using the Maxwell stress tensor, the force \mathbf{F} per unit of length that is exerted by part I on part II (see figure). It can be helpful to make a sketch of the situation, in the xy -plane.

Problem 6 If time is left, you can try to finish problem 3 of last week, and use it to show that:

$$[\mathbf{E}(\nabla \cdot \mathbf{E}) - \mathbf{E} \times (\nabla \times \mathbf{E})]_i = \nabla_j \left(E_i E_j - \frac{1}{2} \delta_{ij} |\mathbf{E}|^2 \right)$$

Some intermediate steps are already given in (7.15) in the syllabus.